FUZZY FREQUENCY RESPONSE FOR UNCERTAIN DYNAMIC SYSTEMS

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Abstract: This paper focuses on the Fuzzy Frequency Response: Definition and Analysis for Uncertain Dynamic Systems. In terms of transfer function, the uncertain dynamic system is partitioned into several linear sub-models and it is organized into Takagi-Sugeno (TS) fuzzy structure. The main contribution of this approach is demonstrated, from a *Theorem*, that fuzzy frequency response is a boundary in the magnitude and phase Bode plots. Low and high frequency analysis of fuzzy dynamic model is obtained by varying the frequency ω from zero to infinity.

1 INTRODUCTION

The design of control systems is currently driven by a large number of requirements posed by increasing competition, environmental requirements, energy and material costs, the demand for robust and fault-tolerant systems. These considerations introduce extra needs for effective process control techniques. In this context, the analysis and synthesis of compensators are completely related to each other. In the analysis, the characteristics or dynamic behaviour of the control system are determined. In the design, the compensators are obtained to attend the desired characteristics of the control system from certain performance criteria. Generally, these criteria may involve disturbance rejection, steady-state erros, transient response characteristics and sensitivity to parameter changes in the plant.

Test input signals is one way to analyse the dynamic behaviour of real world system. Many test signals are available, but a simple and useful signal is the sinusoidal wave form because the system output with a sinusoidal wave input is also a sinusoidal wave, but with a different amplitude and phase for a given frequency. This frequency response analysis describes how a dynamic system responds to sinusoidal inputs in a range of frequencies and has been widely used in academy, industry and considered essential for robust control theory (Serra and Ferreira, 2010).

The frequency response methods were developed during the period 1930 – 1940 by Harry Nyquist (1889 – 1976) (Nyquist, 1932), Hendrik Bode (1905 – 1982) (Bode, 1940), Nathaniel B. Nichols (1914 – 1997) (James et al., 1947) and many others. Since, frequency response methods are among the most useful techniques and available to analyse and synthesise the compensators. In (Jr, 1973), the U.S. Navy obtains frequency responses for aircraft by applying sinusoidal inputs to the autopilots and measuring the resulting position of the aircraft while the aircraft is in flight. In (Lascu et al., 2009), four current controllers for selective harmonic compensation in parallel Active Power Filters (APFs) have been compared analytically in terms of frequency response characteristics and maximum operational frequency. Most real systems, such as circuit components (inductor, resistor, operational amplifier, etc.) are often formulated using differential/integral equations with uncertain parameters (Kolev, 1993). The uncertain about the systems arises from aging, temperature variations, etc. These variations do not follow any of the known probability distributions and are most often quantified in terms of boundaries. The classical methods of frequency response do not explore these boundaries for uncertain dynamic systems. To overcome this limitation, this paper proposes the definition of Fuzzy Frequency Response (FFR) and its application for analysis of uncertain dynamic systems.

2 FORMULATION PROBLEM

This section presents some essentials concepts for the formulation and development of this paper *Fuzzy Frequency Response for Uncertain Dynamic Systems.*

2.1 Uncertain Dynamic Systems

In terms of transfer function, the general form of an uncertain dynamic systems is given by Eq. 1, as depicted in Fig.1.

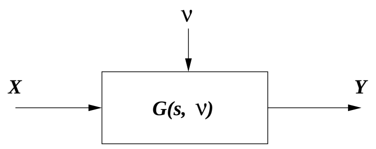

Figure 1: TS fuzzy model

$$G(s,\nu) = \frac{Y(s,\nu)}{X(s)} = \frac{b_\alpha(\nu)s^\alpha + b_{\alpha-1}(\nu)^{\alpha-1} + \ldots + b_\alpha(\nu)s^\alpha + b_1(\nu)s + b_0(\nu)}{s^\beta + a_{\beta-1}(\nu)s^{\beta-1} + \ldots + a_1(\nu)s + a_0(\nu)} \quad (1)$$

where: $X(s)$ and $Y(s,\nu)$ represents the input and the output of uncertain dynamic systems; $a_*(\nu)$ and $b_*(\nu)$ are the varying parameters; $\nu(t)$ is the time varying scheduling variable; s is the Laplace operator; α and β are the orders of the numerator and denominator of the transfer function, respectively (with $\beta \geq \alpha$). The scheduling variable ν belongs to a compact set $\nu \in V$, with its variation limited by $|\dot{\nu}| \leq d^{\max}$, with $d^{\max} \geq 0$. This formulation is very efficient and the fuzzy frequency response of (1) can be used for stability analysis and robust control design.

2.2 Takagi-Sugeno Fuzzy Dynamic Model

The inference system TS, originally proposed in (Takagi and Sugeno, 1985), presents in the consequent a dynamic functional expression of the linguistic variables of the antecedent. The $i\Big|^{[i=1,2,\ldots,l]}$-th rule, where l is the rules numbers, is given by

$Rule^{(i)}$:

$$IF\ \tilde{x}_1 \text{ is } F^i_{\{1,2,\ldots,p_{\tilde{x}_1}\}|\tilde{x}_1}\ AND\ldots AND\ \tilde{x}_n \text{ is } F^i_{\{1,2,\ldots,p_{\tilde{x}_n}\}|\tilde{x}_n}$$

$$\text{THEN}\ \ y_i = f_i(\tilde{\mathbf{x}}) \quad (2)$$

where the total number of rules is $l = p_{\tilde{x}_1} \times \ldots \times p_{\tilde{x}_n}$. The vector $\tilde{\mathbf{x}} = [\tilde{x}_1, \ldots, \tilde{x}_n]^T \in \Re^n$ containing the linguistics variables of antecedent, where T represents the operator for transpose matrix. Each linguistic variable has its own discourse universe $\mathcal{U}_{\tilde{x}_1}, \ldots, \mathcal{U}_{\tilde{x}_n}$, partitioned by fuzzy sets representing its linguistics terms, respectively. In i-th rule, the variable $\tilde{x}_{\{1,2,\ldots,n\}}$ belongs to the fuzzy set $F^i_{\{\tilde{x}_1,\ldots,\tilde{x}_n\}}$ with a membership degree $\mu^i_{F_{\{\tilde{x}_1,\ldots,\tilde{x}_n\}}}$ defined by a membership function $\mu^i_{\{\tilde{x}_1,\ldots,\tilde{x}_n\}} : \Re \rightarrow [0,1]$, with $\mu^i_{F_{\{\tilde{x}_1,\ldots,\tilde{x}_n\}}} \in \{\mu^i_{F_{1|\{\tilde{x}_1,\ldots,\tilde{x}_n\}}}, \mu^i_{F_{2|\{\tilde{x}_1,\ldots,\tilde{x}_n\}}}, \ldots, \mu^i_{F_{p|\{\tilde{x}_1,\ldots,\tilde{x}_n\}}}\}$, where $p_{\{\tilde{x}_1,\ldots,\tilde{x}_n\}}$ is the partitions number of the discourse universe associated with the linguistic variable $\tilde{x}_1, \ldots, ,\tilde{x}_n$. The output of the TS fuzzy dynamic model is a convex combination of the dynamic functional expressions of consequent $f_i(\tilde{\mathbf{x}})$, without lost of generality for the bidimensional case, as illustrated in Fig. 2, given by Eq. 3.

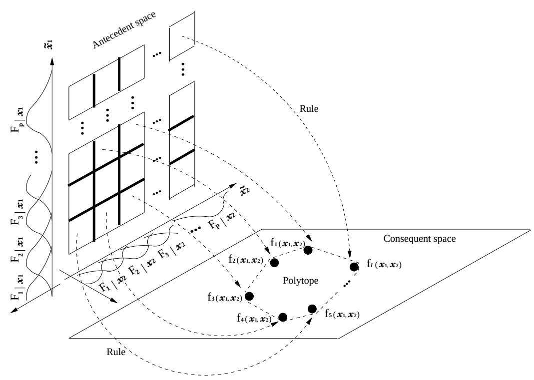

Figure 2: Fuzzy dynamic model: A TS model can be regarded as a mapping from the antecedent space to the space of the consequent parameters one.

$$y(\tilde{\mathbf{x}},\gamma) = \sum_{i=1}^{l} \gamma_i(\tilde{\mathbf{x}}) f_i(\tilde{\mathbf{x}}) \quad (3)$$

where γ is the scheduling variable of the TS fuzzy dynamic model. It can be observed that the TS fuzzy dynamic system, which represents any uncertain dynamic model, may be considered as a class of systems where $\gamma_i(\tilde{\mathbf{x}})$ denotes a decomposition of linguistic variables $[\tilde{x}_1, \ldots, \tilde{x}_n]^T \in \Re^n$ for a polytopic geometric region in the consequent space from the functional expressions $f_i(\tilde{\mathbf{x}})$.

3 FUZZY FREQUENCY RESPONSE (FFR): DEFINITION

This section will present how a TS fuzzy model of an uncertain dynamic system responds to sinusoidal inputs, which in this paper is proposed as the definition of fuzzy frequency response. The response of a TS fuzzy model to a sinusoidal input of frequency ω_1 in both amplitude and phase, is given by the transfer function evaluated at $s = j\omega_1$, as illustrated in Fig. 3.

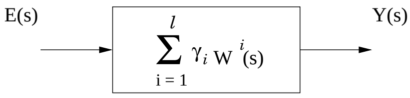

Figure 3: TS fuzzy transfer function.

For this TS fuzzy model,

$$Y(s) = \left[\sum_{i=1}^{l} \gamma_i W^i(s)\right] E(s) \tag{4}$$

Consider $\sum_{i=1}^{l} \gamma_i W^i(j\omega)$ a complex number for a given ω, as

$$\sum_{i=1}^{l} \gamma_i W^i(j\omega) =$$
$$= \left|\sum_{i=1}^{l} \gamma_i W^i(j\omega)\right| e^{j\phi(\omega)} = \left|\sum_{i=1}^{l} \gamma_i W^i(j\omega)\right| \angle\phi(\omega) =$$
$$= \left|\sum_{i=1}^{l} \gamma_i W^i(j\omega)\right| \angle \arctan\left[\sum_{i=1}^{l} \gamma_i W^i(j\omega)\right] \tag{5}$$

Then, for the case that the input signal $e(t)$ is sinusoidal, that is,

$$e(t) = A \sin \omega_1 t \tag{6}$$

the output signal $y_{ss}(t)$, in the steady state, is given by

$$y_{ss}(t) = A \left|\sum_{i=1}^{l} \gamma_i W^i(j\omega)\right| \sin\left[\omega_1 t + \phi(\omega_1)\right] \tag{7}$$

As result of the fuzzy frequency response definition, it is proposed the following theorem:

Theorem 3.1 *Fuzzy frequency response is a region in the frequency domain, defined by the consequent sub-models and from the operating region of the antecedent space.*

Proof Considering that the parameter $\nu(t)$ is uncertain and can be represented by linguistic terms, once known its discurse universe, as shown in Fig. 4, the activation degrees, $h_i(\tilde{\nu})|^{i=1,2,\ldots,l}$, are also uncertain, since it dependes of the dynamic system:

$$h_i(\tilde{\nu}) = \mu^i_{F_{\tilde{\nu}_1^*}} \star \mu^i_{F_{\tilde{\nu}_2^*}} \star \ldots \star \mu^i_{F_{\tilde{\nu}_n^*}} \tag{8}$$

where $\tilde{\nu}^*_{\{1,2,\ldots,n\}} \in \mathcal{U}_{\tilde{\nu}_{\{1,2,\ldots,n\}}}$, respectively, and $\star$ is a fuzzy logic operator.

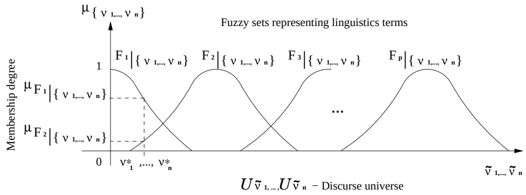

Figure 4: Functional description of the linguistic variables: linguistic terms, discurse universes and membership degrees.

So, the normalized activation degrees $\gamma_i(\nu)|^{i=1,2,\ldots,l}$, are also uncertain, as shown in:

$$\gamma_i(\tilde{\nu}) = \frac{h_i(\tilde{\nu})}{\sum_{r=1}^{l} h_r(\tilde{\nu})} \tag{9}$$

This normalization implies

$$\sum_{k=1}^{l} \gamma_i(\tilde{\nu}) = 1 \tag{10}$$

The output of the TS fuzzy model is a weighted sum of the consequent functional expression, e.g., a linear convex combination of the local functions $f_i(\tilde{\nu})$, and is given by

$$y(\tilde{\nu}) = \sum_{i=1}^{l} \gamma_i(\tilde{\nu}) f_i(\tilde{\nu}) \tag{11}$$

Let $F(\tilde{\nu})$ a vectorial space of transfer functions with degree $\leq l$ and $f^1(s), f^2(s), \ldots, f^l(s)$ transfer functions which belongs to this vectorial space. A transfer function $f(s) \in F(\tilde{\nu})$ must be a linear convex combination of the vectors $f^1(s), f^2(s), \ldots, f^l(s)$. So

$$f(s) = \gamma_1 f^1(s) + \gamma_2 f^2(s) + \ldots + \gamma_l f^l(s) \tag{12}$$
$$f(s) = \sum_{i=1}^{l} \gamma_i(\tilde{\nu}) f_i(\tilde{\nu}) \tag{13}$$

The TS fuzzy model must attend the polytope property. So, the sum of the normalized activation degree must be equal to 1, as shown in Eq (10). To satisfy this property, each rule must be singly activated. This condition is called boundary condition. In this way, the following results are obtained:

If just the rule 1 is activated, it has $(\gamma_1 = 1, \gamma_2 = 0, \gamma_3 = 0, \ldots, \gamma_l = 0)$. Hence,

$$f(s) \quad = \quad 1f^1(s) + 0f^2(s) + \ldots + 0f^l(s) = f^1(s) \tag{14}$$

From (5), it has

$$f(j\omega) = \left|f^1(j\omega)\right| \angle f^1(j\omega) \tag{15}$$

If just the rule 2 is activated, it has $(\gamma_1 = 0, \gamma_2 = 1, \gamma_3 = 0, \ldots, \gamma_l = 0)$. Hence,

$$f(s) \quad = \quad 0f^1(s) + 1f^2(s) + \ldots + 0f^l(s) = f^2(s) \tag{16}$$

From (5), it has

$$f(j\omega) = \left|f^2(j\omega)\right| \angle f^2(j\omega) \tag{17}$$

If just the rule l is activated, it has $(\gamma_1 = 0, \gamma_2 = 0, \gamma_3 = 0, \ldots, \gamma_l = 1)$. Hence,

$$f(s) \quad = \quad 0f^1(s) + 0f^2(s) + \ldots + 1f^l(s) = f^l(s) \tag{18}$$

From (5), it has

$$f(j\omega) = \left|f^l(j\omega)\right| \angle f^l(j\omega) \tag{19}$$

Note that $\left|f^1(j\omega)\right| \angle f^1(j\omega)$ and $\left|f^l(j\omega)\right| \angle f^l(j\omega)$ define a boundary region. Under such circumstances, it seems plausible that the fuzzy frequency response for uncertain dynamic systems converges to a boundary in the frequency response, as shown in Fig.5.

4 CONCLUSIONS

The Fuzzy Frequency Response: Definition and Analysis for Uncertain Dynamic Systems is proposed in this paper. It was shown that the fuzzy frequency response is a region in the frequency domain, defined by the consequent linear sub-models $G^i(s)$, from operating regions of the uncertain dynamic system, according to the proposed *Theorem 3.1*. This formulation is very efficient and can be used for robust stability analysis and control design for uncertain dynamic systems.

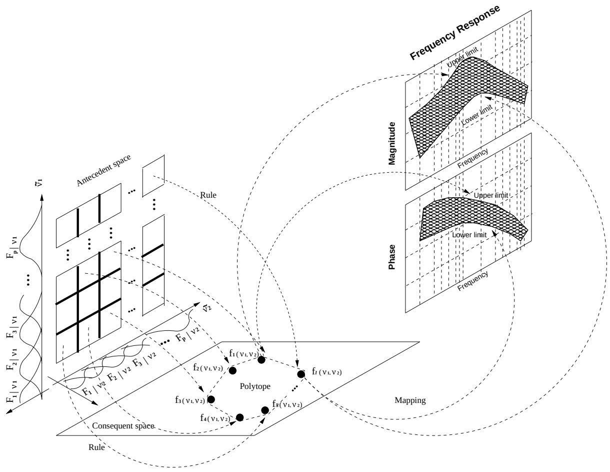

Figure 5: Fuzzy frequency response: mapping from the consequent space to the region in the frequency domain.

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REFERENCES

Bode, H. W. (1940). Feedback amplifier design. *Bell Systems Technical Journal*, 19:42.

James, H. M., Nichols, N. B., and Phillips, R. S. (1947). *Theory of servomechanisms*. McGraw-Hill, MIT Radiation Laboratory Series. New York.

Jr, A. P. S. (1973). Determination of aircraft response caracteristics in approach/landing configuration for microwave landing system program. In *Report FT-61R-73, Naval Air Test Center*. Patuxent River, MD.

Kolev, L. V. (1993). Interval methodsfor circuit analysis. *Singapore: World Scienhfic*.

Lascu, C., Asiminoaei, L., Boldea, I., and Blaabjerg, F. (2009). Frequency response analysis of current controllers for selective harmonic compensation in active power filters. *IEEE Transactions on Industrial Eletronics*, 56(2).

Nyquist, H. (1932). Regeneration theory. *Bell Systems Technical Journal*.

Serra, G. L. O. and Ferreira, C. C. T. (2010). Fuzzy frequncy response: Definition and analysis for nonlinear dynamic systems. *IEEE International Symposium on Industrial Electronics*.

Takagi, T. and Sugeno, M. (1985). Fuzzy identification of systems and its applications to modeling and control. *IEEE Trans. Syst. Man. Cyber*, 15:116–132.