Fuzzy Methodology for Frequency Response Estimation of Nonlinear Dynamic Systems

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***Abstract*—A Fuzzy Methodology for Frequency Response Estimation of Nonlinear Dynamic Systems is proposed in this paper. In terms of transfer function, the nonlinear dynamic system is partitioned into several linear sub-models and it is organized into Takagi-Sugeno (TS) fuzzy structure. The main contribution of this approach is demonstrated, from the proposal of a *Theorem*, that fuzzy frequency response is a boundary in the magnitude and phase Bode plots and useful for estimation of the frequency response of a one-link robotic manipulator. Low and high frequency analysis of the fuzzy dynamic model is obtained by varying the frequency ω from zero to infinity.**

I. Introduction

The design of control systems is currently driven by a large number of requirements posed by increasing competition, environmental requirements, energy and material costs, the demand for robust and fault-tolerant systems. These considerations introduce extra needs for effective process control techniques. In this context, the analysis and synthesis of compensators are completely related to each other. In the analysis, the characteristics or dynamic behaviour of the control system are determined. In the design, the compensators are obtained to attend the desired characteristics of the control system from certain performance criteria. Generally, these criteria may involve disturbance rejection, steady-state errors, transient response characteristics and sensitivity to parameter changes in the plant [3], [4], [5].

Test input signals is one way to analyse the dynamic behaviour of real world system. Many test signals are available, but a simple and useful signal is the sinusoidal wave form because the system output with a sinusoidal wave input is also a sinusoidal wave, but with a different amplitude and phase for a given frequency. This frequency response analysis describes how a dynamic system responds to sinusoidal inputs in a range of frequencies and has been widely used in academy, industry and considered essential for robust control theory [10].

The frequency response methods were developed during the period $1930-1940$ by Harry Nyquist $(1889-1976)$ [9], Hendrik Bode $(1905-1982)$ [2], Nathaniel B. Nichols $(1914-1997)$ [8] and many others. Since, frequency response methods are among the most useful techniques and available to analyse and synthesise the compensators. In [1], the U.S. Navy obtains frequency responses for aircraft by applying sinusoidal inputs to the autopilots and measuring the resulting position of the aircraft while the aircraft is in flight. In [6], four current controllers for selective harmonic compensation in parallel Active Power Filters (APFs) have been compared analytically in terms of frequency response characteristics and maximum operational frequency.

Most real systems, such as circuit components (inductor, resistor, operational amplifier, etc.) are often formulated using differential/integral equations with nonlinearities and uncertain parameters [7]. The nonlinearity about the systems arises from aging, temperature variations, etc. These variations do not follow any of the known probability distributions and are most often quantified in terms of boundaries. The classical methods of frequency response do not explore these boundaries for nonlinear dynamic systems. To overcome this limitation, this paper proposes the definition of Fuzzy Frequency Response (FFR) and its application for analysis of nonlinear dynamic systems.

II. Formulation Problem

This section presents some essentials concepts for the formulation and development of this paper.

A. Takagi-Sugeno Fuzzy Dynamic Model

The inference system TS, originally proposed in [11], presents in the consequent a dynamic functional expression of the linguistic variables of the antecedent. The $i\Big|^{[i=1,2,\ldots,l]}$-th rule, where l is the rules numbers, is given by

$$Rule^{(i)}:$$
$$IF\ \tilde{x}_1 \text{ is } F^i_{\{1,2,\ldots,p_{\tilde{x}_1}\}|\tilde{x}_1}\ AND \ldots AND\ \tilde{x}_n \text{ is } F^i_{\{1,2,\ldots,p_{\tilde{x}_n}\}|\tilde{x}_n}$$
$$\text{THEN}\ \ y_i = f_i(\tilde{\mathbf{x}}) \tag{1}$$

where the total number of rules is $l = p_{\tilde{x}_1} \times \ldots \times p_{\tilde{x}_n}$. The vector $\tilde{\mathbf{x}} = [\tilde{x}_1, \ldots, \tilde{x}_n]^T \in \Re^n$ containing the linguistics variables of antecedent, where T represents the operator for transpose matrix. Each linguistic variable has its own discourse universe $\mathcal{U}_{\tilde{x}_1}, \ldots, \mathcal{U}_{\tilde{x}_n}$, partitioned by fuzzy sets representing its linguistics terms, respectively. In i-th rule, the variable $\tilde{x}_{\{1,2,\ldots,n\}}$ belongs to the fuzzy set $F^i_{\{\tilde{x}_1,\ldots,\tilde{x}_n\}}$ with a membership degree $\mu^i_{F_{\{\tilde{x}_1,\ldots,\tilde{x}_n\}}}$ defined by a membership function $\mu^i_{\{\tilde{x}_1,\ldots,\tilde{x}_n\}} : \Re \to [0,1]$, with $\mu^i_{F_{\{\tilde{x}_1,\ldots,\tilde{x}_n\}}} \in \{\mu^i_{F_{1|\{\tilde{x}_1,\ldots,\tilde{x}_n\}}}, \mu^i_{F_{2|\{\tilde{x}_1,\ldots,\tilde{x}_n\}}}, \ldots, \mu^i_{F_{p|\{\tilde{x}_1,\ldots,\tilde{x}_n\}}}\}$, where

$p_{\{\tilde{x}_1,\ldots,\tilde{x}_n\}}$ is the partitions number of the discourse universe associated with the linguistic variable $\tilde{x}_1, \ldots, , \tilde{x}_n$. The output of the TS fuzzy dynamic model is a convex combination of the dynamic functional expressions of consequent $f_i(\tilde{\mathbf{x}})$ given by:

$$y(\tilde{\mathbf{x}}, \gamma) = \sum_{i=1}^{l} \gamma_i(\tilde{\mathbf{x}}) f_i(\tilde{\mathbf{x}}) \quad (2)$$

where γ is the scheduling variable of the TS fuzzy dynamic model. The scheduling variable, well known as normalized activation degree is given by:

$$\gamma_i(\tilde{\mathbf{x}}) = \frac{h_i(\tilde{\mathbf{x}})}{\sum_{r=1}^{l} h_r(\tilde{\mathbf{x}})}. \quad (3)$$

This normalization implies

$$\sum_{k=1}^{l} \gamma_i(\tilde{\mathbf{x}}) = 1. \quad (4)$$

It can be observed that the TS fuzzy dynamic system, which represents any nonlinear dynamic model, may be considered as a class of systems where $\gamma_i(\tilde{\mathbf{x}})$ denotes a decomposition of linguistic variables $[\tilde{x}_1, \ldots, \tilde{x}_n]^T \in \Re^n$ for a polytopic geometric region in the consequent space from the functional expressions $f_i(\tilde{\mathbf{x}})$.

III. Fuzzy Frequency Response (FFR): Definition

This section will present how a TS fuzzy model of a nonlinear dynamic system responds to sinusoidal inputs, which in this paper is proposed as the definition of fuzzy frequency response. The response of a TS fuzzy model to a sinusoidal input of frequency ω_1 in both amplitude and phase, is given by the transfer function evaluated at $s = j\omega_1$, as illustrated in Fig. 1.

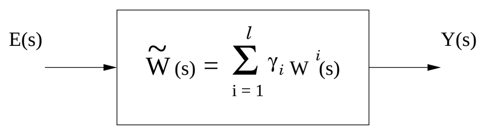

Fig. 1. TS fuzzy transfer function

For this TS fuzzy model,

$$Y(s) = \left[\sum_{i=1}^{l} \gamma_i W^i(s) \right] E(s). \quad (5)$$

Consider $\tilde{W}(j\omega) = \sum_{i=1}^{l} \gamma_i W^i(j\omega)$ a complex number for a given ω, as

$$\tilde{W}(j\omega) = \left| \sum_{i=1}^{l} \gamma_i W^i(j\omega) \right| \angle \arctan \left[\sum_{i=1}^{l} \gamma_i W^i(j\omega) \right]. \quad (6)$$

Then, for the case that the input signal $e(t)$ is sinusoidal, that is,

$$e(t) = A \sin \omega_1 t. \quad (7)$$

The output signal $y_{ss}(t)$, in the steady state, is given by

$$y_{ss}(t) = A \left| \sum_{i=1}^{l} \gamma_i W^i(j\omega) \right| \sin \left[\omega_1 t + \phi(\omega_1) \right]. \quad (8)$$

As result of the fuzzy frequency response definition, shown in Eq. (5)-(8), it is proposed the following *Theorem*:

Theorem 3.1: Fuzzy frequency response is a region in the frequency domain, defined by the consequent sub-models and from the operating region of the antecedent space.

Proof:

Considering that $\tilde{\nu}$ is the linguistic variable of the operation point ν, it can be represented by linguistic terms. Once known its discourse universe, as shown in Fig. 2, the activation degrees $h_i(\tilde{\nu})|^{i=1,2,\ldots,l}$ dependes of the dynamic system and is given by:

$$h_i(\tilde{\nu}) = \mu^i_{F_{\tilde{\nu}^*_1}} \star \mu^i_{F_{\tilde{\nu}^*_2}} \star \ldots \star \mu^i_{F_{\tilde{\nu}^*_n}}, \quad (9)$$

where $\tilde{\nu}^*_{\{1,2,\ldots,n\}} \in \mathcal{U}_{\tilde{\nu}_{\{1,2,\ldots,n\}}}$, respectively, and $\star$ is a fuzzy logic operator.

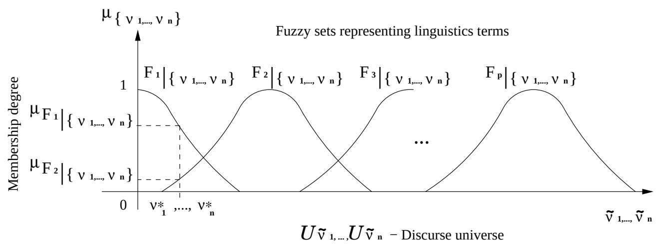

Fig. 2. Functional description of the linguistic variable $\tilde{\nu}$: linguistic terms, discourse universes and membership degrees.

The normalized activation degrees $\gamma_i(\tilde{\nu})|^{i=1,2,\ldots,l}$, are also uncertain:

$$\gamma_i(\tilde{\nu}) = \frac{h_i(\tilde{\nu})}{\sum_{r=1}^{l} h_r(\tilde{\nu})}. \quad (10)$$

This normalization implies

$$\sum_{k=1}^{l} \gamma_i(\tilde{\nu}) = 1. \quad (11)$$

Let $F(s)$ be a vectorial space with degree l and $f^1(s), f^2(s), \ldots, f^l(s)$ transfer functions which belongs to this vectorial space. A transfer function $f(s) \in F(s)$ must be a linear convex combination of the vectors $f^1(s), f^2(s), \ldots, f^l(s)$:

$$f(s) \quad = \quad \xi_1 f^1(s) + \xi_2 f^2(s) + \ldots + \xi_l f^l(s), \qquad (12)$$

where $\xi_{1,2,\ldots,l}$ are the coefficients of this linear convex combination. If the coefficients of the linear convex combination are normalized $\left(\sum_{i=1}^{l} \xi_i = 1\right)$, the vectorial space presents a decomposition of the transfer functions $\left[f^1(s), f^2(s), \ldots, f^l(s)\right]$ in a polytopic geometric shape of the vectorial space $F(s)$. The points of the polytopic geometric shape are defined by the transfer functions $\left[f^1(s), f^2(s), \ldots, f^l(s)\right]$. The TS fuzzy dynamic model attends this polytopic property. The sum of the normalized activation degrees is equal to 1, as shown in Eq. (4). To define the points of this fuzzy polytopic geometric shape, each rule of the TS fuzzy dynamic model must be singly activated. This condition is called boundary condition. In this way, the following results are obtained for the Fuzzy Frequency Response (FFR) of the TS fuzzy transfer function:

- *If only the rule* 1 *is activated, it has* $(\gamma_1 = 1, \gamma_2 = 0, \gamma_3 = 0, \ldots, \gamma_l = 0)$*:*

$$\tilde{W}(j\omega, \tilde{\nu}) = \left| \sum_{i=1}^{l} \gamma_i(\tilde{\nu}) W^i(j\omega) \right| \angle \arctan \left[\sum_{i=1}^{l} \gamma_i(\tilde{\nu}) W^i(j\omega) \right], \qquad (13)$$

$$\tilde{W}(j\omega, \tilde{\nu}) = \left| 1W^1(j\omega) + 0W^2(j\omega) + \ldots + 0W^l(j\omega) \right| \angle \arctan \left[1W^1(j\omega) + 0W^2(j\omega) + \ldots + 0W^l(j\omega) \right],$$

$$\tilde{W}(j\omega, \tilde{\nu}) = \left| W^1(j\omega) \right| \angle \arctan \left[W^1(j\omega) \right]. \qquad (14)$$

- *If only the rule* 2 *is activated, it has* $(\gamma_1 = 0, \gamma_2 = 1, \gamma_3 = 0, \ldots, \gamma_l = 0)$*:*

$$\tilde{W}(j\omega, \tilde{\nu}) = \left| \sum_{i=1}^{l} \gamma_i(\tilde{\nu}) W^i(j\omega) \right| \angle \arctan \left[\sum_{i=1}^{l} \gamma_i(\tilde{\nu}) W^i(j\omega) \right], \qquad (15)$$

$$\tilde{W}(j\omega, \tilde{\nu}) = \left| 0W^1(j\omega) + 1W^2(j\omega) + \ldots + 0W^l(j\omega) \right| \angle \arctan \left[0W^1(j\omega) + 1W^2(j\omega) + \ldots + 0W^l(j\omega) \right],$$

$$\tilde{W}(j\omega, \tilde{\nu}) = \left| W^2(j\omega) \right| \angle \arctan \left[W^2(j\omega) \right]. \qquad (16)$$

- *If only the rule* l *is activated, it has* $(\gamma_1 = 0, \gamma_2 = 0, \gamma_3 = 0, \ldots, \gamma_l = 1)$*:*

$$\tilde{W}(j\omega, \tilde{\nu}) = \left| \sum_{i=1}^{l} \gamma_i(\tilde{\nu}) W^i(j\omega) \right| \angle \arctan \left[\sum_{i=1}^{l} \gamma_i(\tilde{\nu}) W^i(j\omega) \right], \qquad (17)$$

$$\tilde{W}(j\omega, \tilde{\nu}) = \left| 0W^1(j\omega) + 0W^2(j\omega) + \ldots + 1W^l(j\omega) \right| \angle \arctan \left[0W^1(j\omega) + 0W^2(j\omega) + \ldots + 1W^l(j\omega) \right],$$

$$\tilde{W}(j\omega, \tilde{\nu}) = \left| W^l(j\omega) \right| \angle \arctan \left[W^l(j\omega) \right], \qquad (18)$$

where $W^1(j\omega), W^2(j\omega), \ldots, W^l(j\omega)$ are the linear submodels of the nonlinear dynamic system.

Note that $\left|W^1(j\omega)\right| \angle \arctan \left[W^1(j\omega)\right]$ and $\left|W^l(j\omega)\right| \angle \arctan \left[W^l(j\omega)\right]$ define a boundary region. Under such circumstances the fuzzy frequency response for nonlinear dynamic systems converges to a boundary in the frequency domain. Figure 3 shows the fuzzy frequency response for the bidimensional case, without loss of generality.

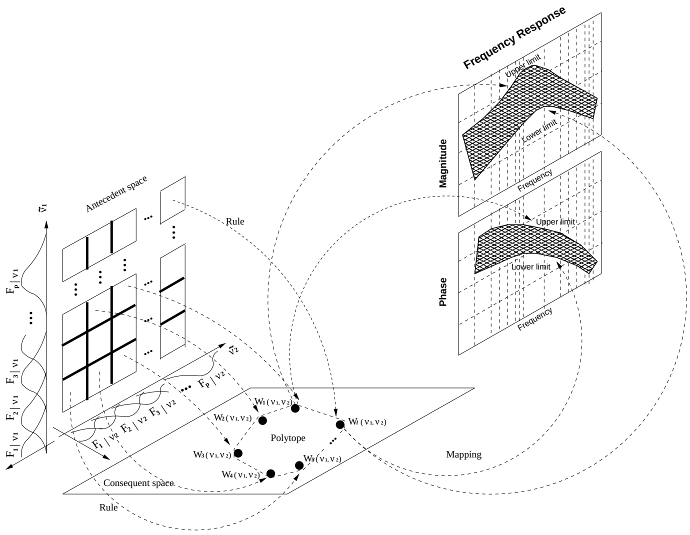

Fig. 3. Fuzzy frequency response: mapping from the consequent space to the region in the frequency domain.

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IV. Fuzzy Frequency Response (FFR): Analysis

A. Low Frequencies Analysis

The low frequencies analysis of the TS fuzzy dynamic model $\tilde{W}(s)$ can be obtained by

$$\lim_{\omega \to 0} \sum_{i=1}^{l} \gamma_i W^i(j\omega). \qquad (19)$$

The magnitude and phase behaviour at low frequencies, is given by

$$\lim_{\omega\to 0}\left|\sum_{i=1}^{l}\gamma_i W^i(j\omega)\right| \angle \arctan\left[\sum_{i=1}^{l}\gamma_i W^i(j\omega)\right]. \quad (20)$$

B. High Frequencies Analysis

Equivalently, the high frequencies analysis of the TS fuzzy dynamic model $\tilde{W}(s)$ can be obtained by

$$\lim_{\omega\to\infty}\sum_{i=1}^{l}\gamma_i W^i(j\omega). \quad (21)$$

The magnitude and phase behaviour at high frequencies, is given by

$$\lim_{\omega\to\infty}\left|\sum_{i=1}^{l}\gamma_i W^i(j\omega)\right| \angle \arctan\left[\sum_{i=1}^{l}\gamma_i W^i(j\omega)\right]. \quad (22)$$

V. Computational Results

To illustrate the FFR: definition and analysis, as shown in section III and IV, it was used the one-link robotic manipulator shown in Fig. 4. The dynamic equation of the one-link robotic manipulator is given by

$$ml^2\ddot{\theta} + d\dot{\theta} + mgl\sin(\theta) = u, \quad (23)$$

where: $m = 1kg$ is the payload; $l = 1m$ is the length of the link; $g = 9.81m/s^2$ is the gravitational constant; $d = 1kgm^2/s$ is the damping factor; u is the control variable (kgm^2/s^2).

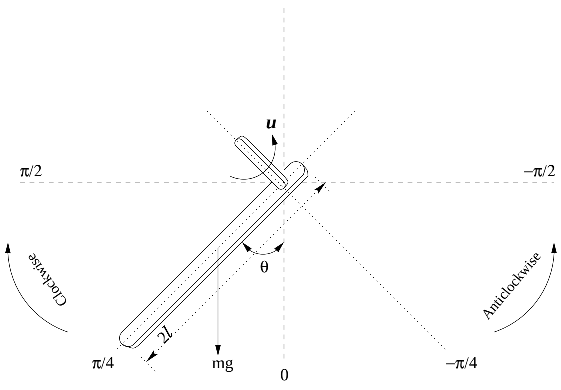

Fig. 4. One-link robotic manipulator.

A LPV model can be obtained from nonlinear model in the Eq. (23) by Taylor series expansion of the nonlinearity $\sin\theta$ in some operating points. For the case that ν is close to ν_0, it can be able to ignore the higher-order derivative terms. Thus

$$f(\nu) \cong f(\nu_0) + \left.\frac{df(\nu)}{d\nu}\right|_{\nu=\nu_0}(\nu-\nu_0). \quad (24)$$

From Eq. (24), the LPV Plant is

$$ml^2\ddot{\theta} + d\dot{\theta} + mgl\left[a + b\theta\right] = u, \quad (25)$$

where $a = \sin\nu - \nu\cos\nu$; $b = \cos\nu$ and ν is the scheduling variable that represents the operating point (angle). In terms of transfer function, it has

$$H(s,\nu) = \frac{\Theta(s,\nu)}{U(s,\nu)} = \frac{1}{ml^2s^2 + ds + mgl\cos\nu}, \quad (26)$$

where $U(s,\nu) = U(s) - mgl[\sin\nu - \nu\cos\nu]$. From the LPV model in Eq. (26) and assuming the dynamics of the system in the range of $[-\pi/4, \pi/4]$, it can obtain the TS Fuzzy Model choosing some operating points:

Sub-model 1 ($\nu = \theta = -\pi/4$):

$$W^1(s,-\pi/4) = \frac{\Theta(s,-\pi/4)}{U(s,-\pi/4)} = \frac{1}{s^2+s+6.9367}. \quad (27)$$

Sub-model 2 ($\nu = \theta = 0$):

$$W^2(s,0) = \frac{\Theta(s,0)}{U(s,0)} = \frac{1}{s^2+s+9.81}. \quad (28)$$

Sub-model 3 ($\nu = \theta = +\pi/4$):

$$W^3(s,+\pi/4) = \frac{\Theta(s,+\pi/4)}{U(s,+\pi/4)} = \frac{1}{s^2+s+6.9367}. \quad (29)$$

The TS fuzzy dynamic model rules base results

$$\begin{array}{lllllll} Rule^{(1)}: & \text{IF} & \nu & \text{is} & -\pi/4 & \text{THEN} & W^1(s,-\pi/4) \\ Rule^{(2)}: & \text{IF} & \nu & \text{is} & 0 & \text{THEN} & W^2(s,0) \\ Rule^{(3)}: & \text{IF} & \nu & \text{is} & +\pi/4 & \text{THEN} & W^3(s,+\pi/4), \end{array} \quad (30)$$

and the TS fuzzy model of the one-link robotic manipulator is given by

$$\tilde{W}(s,\tilde{\nu}) = \sum_{i=1}^{3}\gamma_i(\tilde{\nu})W^i(s). \quad (31)$$

A comparative analysis, via analog simulation, between the one-link robotic manipulator given by Eq.(23) and the TS fuzzy dynamic model given by Eq.(31) can be performed to validate the TS fuzzy dynamic model. In this analysis, an impulse input was considered. As shown in Fig. 5, it can be seen the efficiency of the TS fuzzy dynamic model to represent the dynamic behaviour of the one-link robotic manipulator in the time domain.

From Eq. (6) the TS fuzzy dynamic model of the one-link robotic manipulator can be represented by

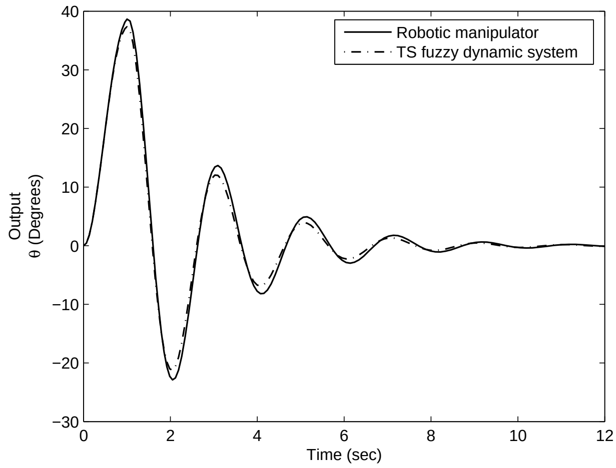

Fig. 5. Validation of the TS fuzzy dynamic model for the one-link robotic manipulator.

$$\tilde{W}(j\omega,\tilde{\nu}) = \left|\sum_{i=1}^{3}\gamma_i(\tilde{\nu})W^i(j\omega)\right| \angle \arctan\left[\sum_{i=1}^{3}\gamma_i(\tilde{\nu})W^i(j\omega)\right] \quad (32)$$

or

$$\tilde{W}(j\omega,\tilde{\nu}) = \left|\gamma_1 W^1(j\omega,-\pi/4) + \gamma_2 W^2(j\omega,0) + \gamma_3 W^3(j\omega,\pi/4)\right| \angle \arctan\left[\gamma_1 W^1(j\omega,-\pi/4) + \gamma_2 W^2(j\omega,0) + +\gamma_3 W^3(j\omega,\pi/4)\right] \quad (33)$$

and

$$\tilde{W}(j\omega,\tilde{\nu}) = = \left|\frac{\gamma_1}{(j\omega)^2+(j\omega)+6.9367} + \frac{\gamma_2}{(j\omega)^2+(j\omega)+9.81} + \frac{\gamma_3}{(j\omega)^2+(j\omega)+6.9367}\right| \angle \arctan \left\{\frac{\gamma_1}{(j\omega)^2+(j\omega)+6.9367} + \frac{\gamma_2}{(j\omega)^2+(j\omega)+9.81} + \frac{\gamma_3}{(j\omega)^2+(j\omega)+6.9367}\right\} \quad (34)$$

$$\tilde{W}(j\omega,\tilde{\nu}) = \left|\frac{\gamma_1[(j\omega)^2+(j\omega)+9.81] + \gamma_2\left[(j\omega)^2+(j\omega)+6.9367\right]}{Den_{[\tilde{W}(j\omega,\tilde{\nu})]}} + \frac{\gamma_3[(j\omega)^2+(j\omega)+9.81]}{Den_{[\tilde{W}(j\omega,\tilde{\nu})]}}\right| \angle \arctan \left\{\frac{\gamma_1[(j\omega)^2+(j\omega)+9.81] + \gamma_2\left[(j\omega)^2+(j\omega)+6.9367\right]}{Den_{[\tilde{W}(j\omega,\tilde{\nu})]}} + \frac{\gamma_3[(j\omega)^2+(j\omega)+9.81]}{Den_{[\tilde{W}(j\omega,\tilde{\nu})]}}\right\} \quad (35)$$

$$\tilde{W}(j\omega,\tilde{\nu}) = \left|\frac{[\gamma_1+\gamma_2+\gamma_3](j\omega)^2 + [\gamma_1+\gamma_2+\gamma_3](j\omega)+}{Den_{[\tilde{W}(j\omega,\tilde{\nu})]}} + \frac{9.81\gamma_1 + 6.9367\gamma_2 + 9.81\gamma_3}{Den_{[\tilde{W}(j\omega,\tilde{\nu})]}}\right| \angle \arctan \left\{\frac{[\gamma_1+\gamma_2+\gamma_3](j\omega)^2 + [\gamma_1+\gamma_2+\gamma_3](j\omega)+}{Den_{[\tilde{W}(j\omega,\tilde{\nu})]}} + \frac{9.81\gamma_1 + 6.9367\gamma_2 + 9.81\gamma_3}{Den_{[\tilde{W}(j\omega,\tilde{\nu})]}}\right\} \quad (36)$$

where

$$Den_{[\tilde{W}(j\omega,\tilde{\nu})]} = (j\omega)^4 + 2(j\omega)^3 + 17.7467(j\omega)^2 + +16.7467(j\omega) + 68.0490 \quad (37)$$

A. Low Frequencies Analysis

From the TS fuzzy dynamic model, Eq. (32), and applying the concepts seen in the subsection IV-A, the steady-state response for sinusoidal input at low frequencies for the one-link robotic manipulator can be obtained as follow:

$$\lim_{\omega\to 0}\tilde{W}(j\omega,\tilde{\nu}) = \left|\frac{[\gamma_1+\gamma_2+\gamma_3](j\omega)^2 + [\gamma_1+\gamma_2+\gamma_3](j\omega)+}{Den_{[\tilde{W}(j\omega,\tilde{\nu})]}} + \frac{9.81\gamma_1 + 6.9367\gamma_2 + 9.81\gamma_3}{Den_{[\tilde{W}(j\omega,\tilde{\nu})]}}\right| \angle \arctan \left\{\frac{[\gamma_1+\gamma_2+\gamma_3](j\omega)^2 + [\gamma_1+\gamma_2+\gamma_3](j\omega)+}{Den_{[\tilde{W}(j\omega,\tilde{\nu})]}} + \frac{9.81\gamma_1 + 6.9367\gamma_2 + 9.81\gamma_3}{Den_{[\tilde{W}(j\omega,\tilde{\nu})]}}\right\} \quad (38)$$

As ω tends to zero, Eq. (38) can be approximated as follow:

$$\lim_{\omega\to 0}\tilde{W}(j\omega,\tilde{\nu}) = \left|\frac{9.81\gamma_1 + 6.9367\gamma_2 + 9.81\gamma_3}{68.0490}\right| \angle \arctan \left\{\frac{9.81\gamma_1 + 6.9367\gamma_2 + 9.81\gamma_3}{68.0490}\right\} \quad (39)$$

Hence

$$\lim_{\omega \to 0} \tilde{W}(j\omega, \tilde{\nu}) = |0.1442\gamma_1 + 0.1019\gamma_2 + 0.1442\gamma_3| \angle 0^o \tag{40}$$

B. High Frequencies Analysis

Likewise, from the TS fuzzy dynamic model, Eq. (32), and now applying the concepts seen in the subsection IV-B, the steady-state response for sinusoidal input at high frequencies for the one-link robotic manipulator can be obtained as follow:

$$\lim_{\omega \to \infty} \tilde{W}(j\omega, \tilde{\nu}) = \left| \frac{[\gamma_1 + \gamma_2 + \gamma_3](j\omega)^2 + [\gamma_1 + \gamma_2 + \gamma_3](j\omega) +}{Den_{[\tilde{W}(j\omega,\tilde{\nu})]}} + \frac{9.81\gamma_1 + 6.9367\gamma_2 + 9.81\gamma_3}{Den_{[\tilde{W}(j\omega,\tilde{\nu})]}} \right| \angle \arctan \left\{ \frac{[\gamma_1 + \gamma_2 + \gamma_3](j\omega)^2 + [\gamma_1 + \gamma_2 + \gamma_3](j\omega) +}{Den_{[\tilde{W}(j\omega,\tilde{\nu})]}} + \frac{9.81\gamma_1 + 6.9367\gamma_2 + 9.81\gamma_3}{Den_{[\tilde{W}(j\omega,\tilde{\nu})]}} \right\} \tag{41}$$

In this analysis, the higher degree terms of the transfer functions in the TS fuzzy dynamic model increase more rapidly than the other ones. Thus,

$$\lim_{\omega \to \infty} \tilde{W}(j\omega, \tilde{\nu}) = \left| \frac{(\gamma_1 + \gamma_2 + \gamma_3)}{(j\omega)^2} \right| \angle \arctan \left\{ \frac{(\gamma_1 + \gamma_2 + \gamma_3)}{(j\omega)^2} \right\}$$

Hence

$$\lim_{\omega \to \infty} \tilde{W}(j\omega, \tilde{\nu}) = \left| \frac{(\gamma_1 + \gamma_2 + \gamma_3)}{(j\omega)^2} \right| \angle - 180^o$$

Figure 6 shows the fuzzy frequency response characteristics for the one-link robotic manipulator from the proposed methodology.

VI. Conclusion

A fuzzy methodology for frequency response estimation of nonlinear dynamic systems is proposed in this paper. The fuzzy frequency response is a region in the frequency domain, defined by the consequent linear sub-models $W^i(s)$, from operating regions of the nonlinear dynamic system, according to the proposed *Theorem 3.1*. This formulation is very efficient and can be used for robust stability analysis and control design for nonlinear dynamic systems.

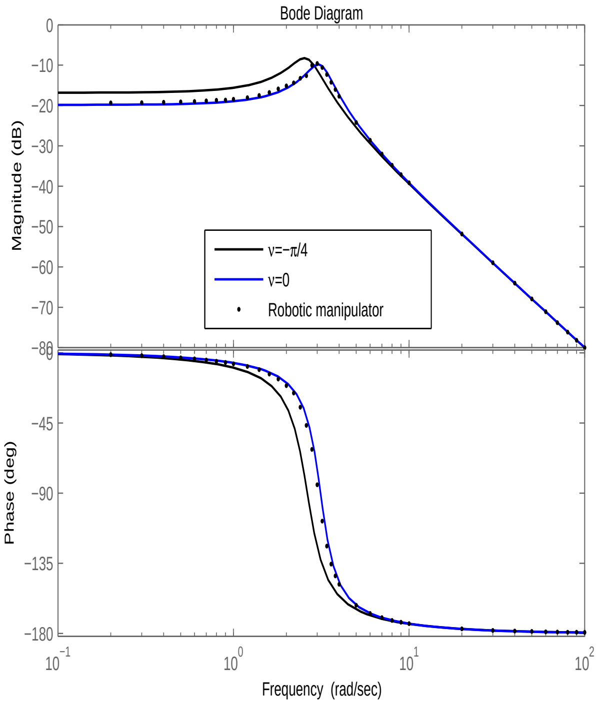

Fig. 6. Fuzzy frequency response characteristics of one-link robotic manipulator.

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